office hr week 13(a)

LAST Name: _____

First Name: _____

Student Number: _____

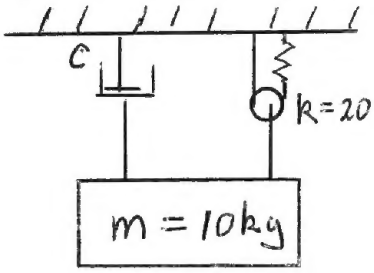
MIE 200F - Quiz number 8b - November 29/01
quiz duration = 25 minutes

A block is supported by the system of dampers and springs as shown. Assume that the mass can move only in the vertical direction, with no rotation. Values of all dampers and springs are given in SI units.

(a) For what values of the damper "C" will the system be underdamped?

(b) What value of damper "C" is required to reduce the magnitude of the oscillations by a factor of 10 within 30 seconds?

$g = 9.81\ m/s^2 \downarrow$



The equation $\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = 0$, has the solution

$x(t) = C_1 \cos(\omega_d t)\exp(-\zeta\omega_n t) + C_2 \sin(\omega_d t)\exp(-\zeta\omega_n t)$

$= C_3 \sin(\omega_d t + \varphi)\exp(-\zeta\omega_n t)$

where $\omega_d = \omega_n (1-\zeta^2)^{1/2}$

$+\downarrow$ (a) $F = -C\dot{x} - (4)(20)x = m\ddot{x}$

$10\ddot{x} + C\dot{x} + 80x = 0$

$\ddot{x} + \frac{C}{10}\dot{x} + 8x = 0 \iff \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 = 0$

$\Rightarrow \omega_n = \sqrt{8}$; $C/10 \sim 2\zeta\omega_n$ $\sqrt{8}$

for critically damped system, $\zeta = 1 \rightarrow C_c = 56.6$ N/m/s

system is underdamped for $C < 56.6$ N/m/s

(b) $\exp\left(-(\zeta)(\omega_n)(30\text{seconds})\right) = 1/10$

take ln of both sides:

$-(\zeta)(\omega_n)(30) = -2.303$

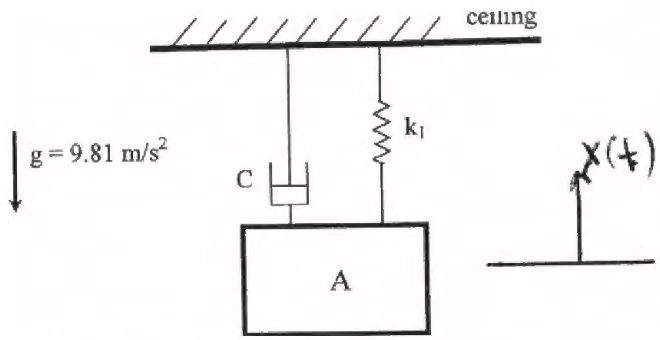
$\omega_n = \sqrt{8}$

0.027 $\sqrt{8}$ $\zeta = 0.027$

$C/10 = 2\zeta\omega_n \Rightarrow C = 1.53$ N/m/s

MIE 100 Final Exam 2008

Block A of mass 50 kg is suspended from the ceiling by a spring with stiffness constant $k_1 = 800$ N/m, and a dashpot of strength C. The mass moves only in the vertical direction, without rotation.



a) What is the period (τ) of natural vibration of the mass, if C = 0? (6 marks)

b) Suppose that A is vibrating in the vertical direction, with an amplitude of vibration equal to 0.5 m. What is the maximum acceleration of A? Assume that C = 0. (6 marks)

(a) $m\ddot{x} + \cancel{c}^{0}\dot{x} + k_1 x = 0$

$\omega_n = \sqrt{k_1/m} = \sqrt{800/50} = 4\ s^{-1}$

$\tau = 2\pi/\omega_n =$ **1.57 s**

(b) $x(t) = (0.5\ m)\sin(\cancel{\omega_n}^{4} t + \phi)$

$\dot{x}(t) = (0.5\ m)(\cancel{\omega_n}^{4})\cos(\cancel{\omega_n}^{4} t + \phi)$

$V_{max} = (0.5\ m)(4\ s^{-1}) =$ **2 m/s**

MIE 100 Final Exam 2008

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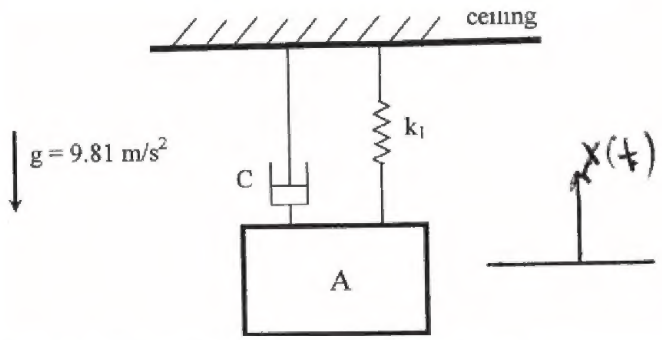


c) What value of C would be required to reduce the amplitude of vibration from 0.5 m to 0.2 m in 15 seconds? (7 marks)

d) Suppose that an earthquake were to make the ceiling vibrate vertically with an amplitude of 3 mm at a frequency of 5 rad/s. What would be the amplitude of vibration of A? Assume C = 0. (6 marks)

(c) $\ddot{x} + \frac{c}{m}\dot{x} + \frac{k_1}{m}x = 0 \quad \longleftrightarrow \quad \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$

$\rightarrow \frac{k_1}{m} = \omega_n^2 \quad , \quad \frac{c}{m} = 2\zeta\omega_n$

$0.2 \text{ meters} = 0.5 \text{ meters} \exp(-\zeta\omega_n t)$

$\ln\left[0.4 = \exp(-\zeta\omega_n t)\right]$

$-0.9163 = -\zeta\omega_n t \quad (\omega_n = 4,\ t = 15) \quad \rightarrow \quad \zeta = 0.0153$

But $\frac{c}{m} = 2\zeta\omega_n \quad (m = 50,\ \zeta = 0.0153,\ \omega_n = 4) \quad \rightarrow \quad \boxed{c = 6.1 \text{ N·s/m}}$

(d) $x(t) = (\delta_0)(M)\sin(\omega_0 t + \phi)$

$= (3\text{ mm})\left(\frac{1}{1 - \omega_0^2/\omega_n^2}\right)(\sin\omega_0 t + \phi)$

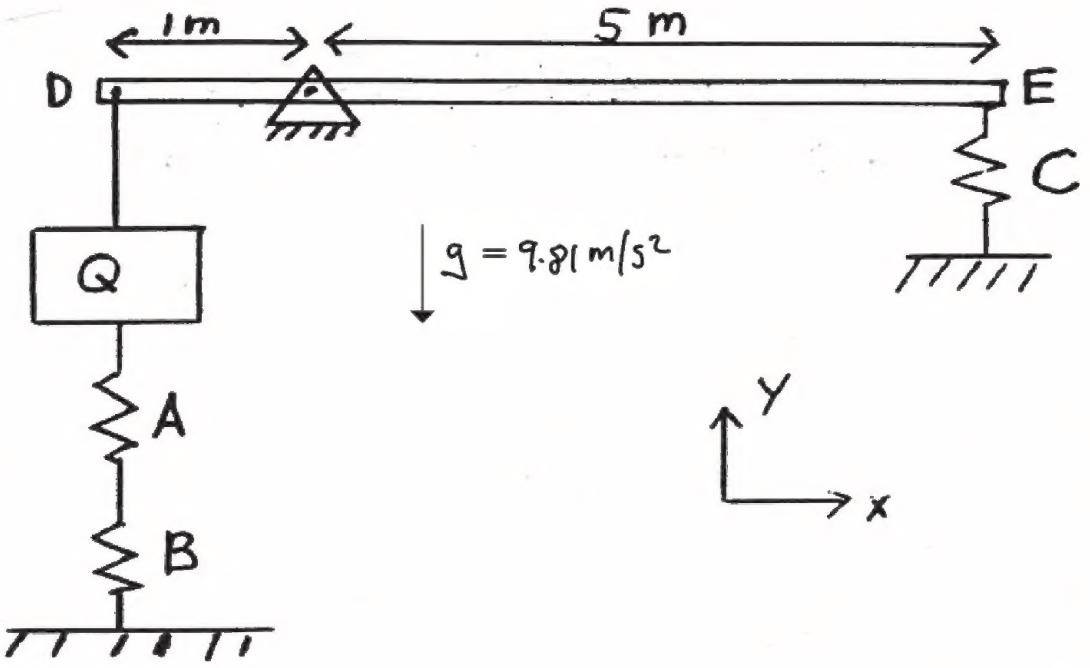
(amplitude)

$\text{amplitude} = (3\text{ mm})\left(\frac{1}{1 - 5^2/4^2}\right) = \boxed{5.33 \text{ mm}}$

MIE 200 Final exam 1998

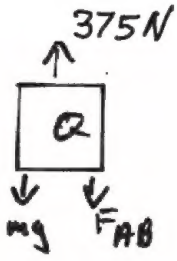
office hour week 13(d)

5. Block "Q" of mass 20 kg is suspended by a system of three springs "A", "B", and "C", plus the weightless bar "DE" which has a length of 6 meters. Each spring has a stiffness constant of 50 N/m. The system is initially at rest with the spring "C" stretched 1.5 meters beyond its relaxed length.
   (a) In the initial static position, by how much is spring "B" stretched beyond its relaxed length?
   (b) Suppose the mass is displaced upwards by +0.01 meters from its initial static position and then released. Determine the sum of all forces acting on the mass at the instant after it is released.



(a) $F_C = (1.5 \text{ meters})(50 \text{ N/m}) = 75 \text{ N}$.

$\Rightarrow$ force at "D" $= \left(\frac{5}{1}\right)(75 \text{ N}) = 375 \text{ N}$.



$\uparrow \Sigma F = 0 = 375 - (20)(9.81) - (50 \text{ N/m})(\text{stretch}_B) = 0$

$\Rightarrow$ equilibrium stretch in springs A & B $= 3.58$ m.

(b) ΣF: springs A & B are equivalent to $\frac{(50)(50)}{50+50} = 25 \text{ N/m}$

Lever multiplies effectiveness of "C" by a factor of $5^2 = 25$

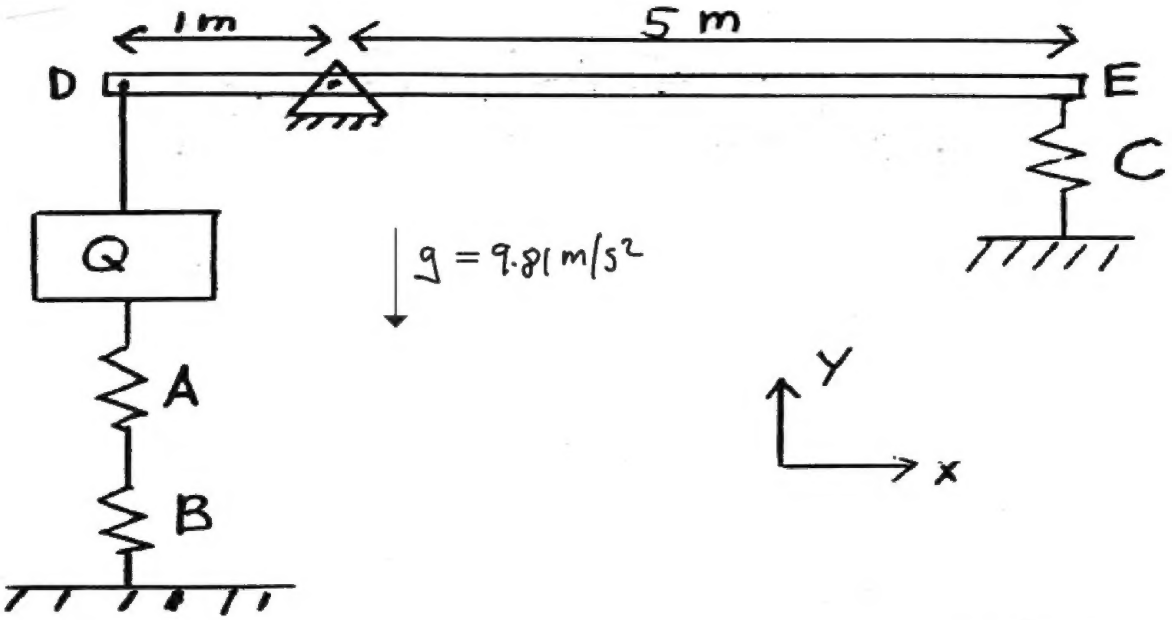
$\Rightarrow$ total $k = 25 + (25)(50) = 1275 \text{ N/m}$

displace 0.01 m $\Rightarrow$ $F = -ky = -(1275)(.01) = \boxed{-12.75 \text{ N}\hat{j}}$

Block "Q" of mass 20 kg is suspended by a system of three springs "A", "B", and "C", plus the weightless bar "DE" which has a length of 6 meters. Each spring has a stiffness constant of 50 N/m. The system is initially at rest with the spring "C" stretched 1.5 meters beyond its relaxed length.

(c) What is the natural frequency ω_n for this system?

(d) A force $F = 2\sin(\omega t)$ Newtons is now applied to block "Q" in the positive y direction. What is the largest value of ω for which the resulting peak-to-peak amplitude of oscillation of block "Q" will be at least 0.03 m?

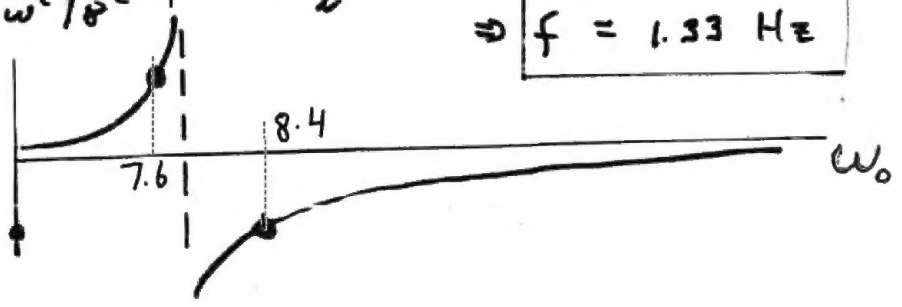


(c) $\omega_n = \sqrt{k/m} = \sqrt{1275/20} = \boxed{8\ s^{-1}}$

(d) $\ddot{x} + \frac{1275}{20} x = 2/20 \sin\omega_0 t$

$\Rightarrow x(t) = \frac{2/1275}{1 - \omega_0^2/8^2} \sin\omega_0 t$

$\left|\frac{2/1275}{1 - \omega^2/8^2}\right| = \frac{0.03}{2} \Rightarrow \omega_0 = 8.4\ s^{-1}$ (or $7.6\ s^{-1}$)

$\Rightarrow \boxed{f = 1.33\ Hz}$

$M = \frac{1}{1 - \omega^2/8^2}$